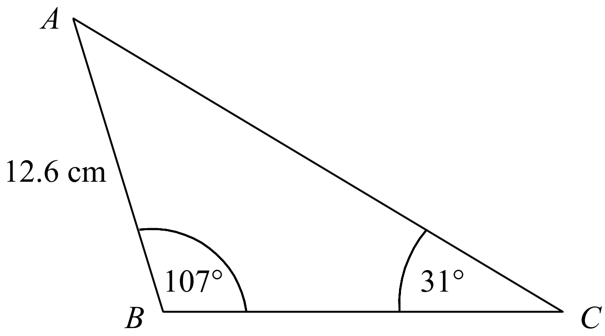
Core Mathematics C2 Paper F

1. Evaluate

$$\sum_{r=1}^{12} (5 \times 2^r). \qquad [4]$$

2.



The diagram shows triangle ABC in which $AB = 12.6$ cm, $\angle ABC = 107°$ and $\angle ACB = 31°$.

Find

(i) the length BC, [3]

(ii) the area of triangle ABC. [2]

3. The curve with equation $y = \mathrm{f}(x)$ passes through the point (8, 7).

Given that

$$\mathrm{f}'(x) = 4x^{\frac{1}{3}} - 5,$$

find $\mathrm{f}(x)$. [6]

4. Solve the equation

$$\sin^2 \theta = 4 \cos \theta,$$

for values of θ in the interval $0 \leq \theta \leq 360°$. Give your answers to 1 decimal place. [7]

5. *(i)* Evaluate

$$\log_3 27 - \log_8 4.$$ [4]

(ii) Solve the equation

$$4^x - 3(2^{x+1}) = 0.$$ [5]

6. *(a)* Expand $(1 + x)^4$ in ascending powers of x. [2]

(b) Using your expansion, express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.

(i) $(1 + \sqrt{2})^4$ [3]

(ii) $(1 - \sqrt{2})^8$ [4]

7. The second and fifth terms of an arithmetic sequence are 26 and 41 repectively.

(i) Show that the common difference is 5. [3]

(ii) Find the 12th term. [3]

Another arithmetic sequence has first term −12 and common difference 7.

Given that the sums of the first n terms of these two sequences are equal,

(iii) find the value of n. [4]

Turn over

8. The polynomial p(x) is defined by

$$p(x) = 2x^3 + x^2 + ax + b,$$

where a and b are constants.

Given that when p(x) is divided by $(x + 2)$ there is a remainder of 20,

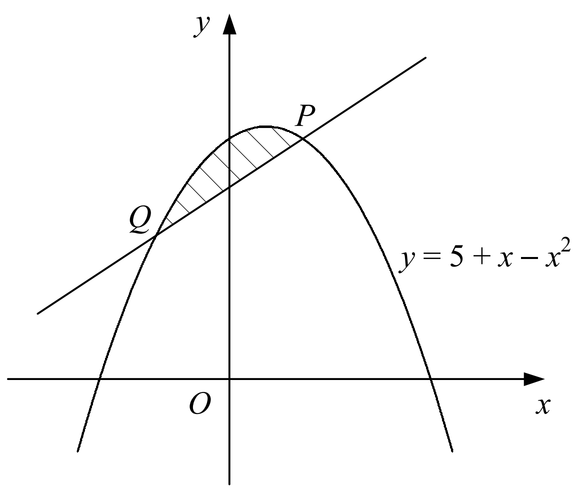
(i) find an expression for b in terms of a. [2]

Given also that $(2x - 1)$ is a factor of p(x),

(ii) find the values of a and b, [4]

(iii) fully factorise p(x). [4]

9.



The diagram shows the curve with equation $y = 5 + x - x^2$ and the normal to the curve at the point P (1, 5).

(i) Find an equation for the normal to the curve at P in the form $y = mx + c$. [5]

(ii) Find the coordinates of the point Q, where the normal to the curve at P intersects the curve again. [2]

(iii) Show that the area of the shaded region bounded by the curve and the straight line PQ is $\frac{4}{3}$. [5]